MATH 54 - HINTS TO HOMEWORK 5

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Here are a couple of hints to Homework 5! Enjoy! :)

Section 4.4: Coordinate Systems

Remember: It's easier to figure out $\mathbf{x}$ once we know $[\mathbf{x}]_{\mathcal{B}}$ than the reverse. Also, remember that the change of coordinates matrix is just the matrix whose columns are the elements in $\mathcal{B}$.

It takes a code as its input and tells you which vector corresponds to that code. Its inverse matrix does what you usually want: It produces the coordinates of $\mathbf{x}$

4.4.27. Use the basis $\mathcal{B} = \left\{1, t, t^2, t^3\right\}$, and compute the coordinates of the 4 polynomials. Then the polynomials are linearly independent if and only if their corresponding vectors are linearly independent!

Section 4.5: The dimension of a vector space

4.5.1, 4.5.3, 4.5.7, 4.5.9. First express the subspace as the span of some vectors, and then use the following useful trick:

Useful trick: To find a basis of a collection of vectors, form the matrix A whose columns are the vectors, and all you need to do is to find a basis for $Col(A)$. In particular, the dimension of the subspace is the dimension of $Col(A)$ (which is the number of pivots).

4.5.26. I did this on Friday! Suppose $\mathcal{B} = \{\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}\}$ is a basis for H. What two things can you say about $\mathcal{B}$? Then use the Basis theorem (Theorem 12).

4.5.27. Find an infinite linearly independent set in $\mathbb{P}$. For example, $\left\{1, x, x^2 \cdots\right\}$ works!

Section 4.6: The rank of a matrix

Remember that the rank of A is just $dim(Col(A))$. It is also equal to $dim(Row(A))$ and to $Rank(A^T)$ and to the number of pivots of A.

4.6.7, 4.6.9, 4.6.11, 4.6.13, 4.6.15. Use the equation $dim(Nul(A)) + Rank(A) = n$. Also, $rank(A)$ is largest when $Nul(A)$ is smallest.

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4.6.17.

(a) **T**
(b) **F**
(c) **T**
(d) **F**
(e) **F**

4.6.23. This question is just meant to confuse you with words! All that it says is if $dim(Nul(A)) = 2$ and $n = 8$, then what is $Row(A)$?

4.6.33. I urge you to do $4.6.32$ before, it makes this much easier! The point is that if A has rank 1, then all its columns are multiples of the first column. In particular, let $\mathbf{v}$ be the list of the coefficients. For example, if $A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix}$, then let $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, because the second column is -3 times the first one and the third column is 4 times the first one.

If the first column of A is zero, try the second column. If the second column is zero, try the third column. If neither of those hold, then A is the zero matrix, which does not have rank 1.

SECTION 4.7: CHANGE OF BASIS

Remember: To change coordinates from $\mathcal{B}$ to $\mathcal{C}$, just express the vectors in $\mathcal{B}$ in terms of the vectors in $\mathcal{C}$

4.7.33. (ii), because P is just $\mathcal{W} \overset{P}{\leftarrow} \mathcal{U}$, so P goes from $\mathcal{U}$ to $\mathcal{V}$.

4.7.11.

(a) **F**
(b) **T**

4.7.17. IGNORE! But if you do $4.7.17$ and $4.7.18$, you'll have a completely different and awesome view of calculus and linear algebra!